

cathode-ray oscilloscope. This mask is completely opaque; light can only pass through two narrow slots, one representing the  $x$ -axis and the second the function  $f(x)$ . The deflection of the beam in the horizontal direction is controlled by the voltage  $g(t)$ . The vertical deflection plates are energized by a sawtooth voltage having a frequency considerably greater than that of  $g(t)$ . Accordingly, the beam traverses a large number of times the distance between the  $x$ -axis and  $f(x)$  in the course of a single horizontal scan. A photoelectric system is placed in front of the oscillograph to produce an electrical impulse each time the beam passes the two slots. The interval between the two impulses in the course of a single scanning period has the form

$$\tau_n = f[g(t_n)]$$

The impulses thus obtained trigger a multivibrator which produces rectangular voltage pulses whose width is proportional to  $\tau_n$ . The output of the multivibrator controls an electronic switch which in turn acts to interrupt the second variable voltage  $v(t)$ . The average value of the output current is therefore of the form of  $e_o$  in Equation 6.37.

In order to prepare the mask, the function is drawn in indian ink on drafting paper (the line thickness is 0.8 mm), and the drawing is reproduced photographically at a 6:1 scale. The dimensions of the mask on the screen are  $82 \times 82$  mm. The static accuracy of this generator is 0.5%; the bandwidth is 10 cycles/sec; and the dynamic range of the signals at the input and output is  $\pm 100$  volts. With certain modifications this generator can be adapted to function generation in all four quadrants. Polimerou<sup>33</sup> has proposed the extension of the photoformer technique to the described method.

Another approach to nonlinear function generation, described by Savant,<sup>34</sup> employs the relationship

$$y = [f(x)]^a [g(y)]^b [h(z)]^c \quad (6.40)$$

Taking logarithms of both sides of the above expression yields

$$\log y = a \log f(x) + b \log g(y) + c \log h(z) \quad (6.41)$$

Since addition and subtraction are readily carried out using standard analog units, the basic problem is the construction of an element that generates the logarithm of a function and the inverse logarithm of a function. Such a transformation element was described by Howard.<sup>35</sup>

## 6.9 Generators of Functions of Two Variables

Occasionally in analog computations it is desired to generate functions that are themselves functions of two or more dependent variables. Sometimes the pertinent functional relationships are available in analytic form,

at other times the functions are arbitrary and presented in graphical form. Examples of the latter condition are the familiar vacuum tube curves. Over the years a wide variety of devices to generate such functions have been suggested. Only a few of these, however, have sufficiently large bandwidths to make them suitable for repetitive operation. The more promising of these methods are described here briefly.

Basically the problem involves the generation of

$$\begin{aligned} z &= f(x, y) \\ a &\leq x \leq b \\ c &\leq y \leq d \end{aligned} \quad (6.42)$$

The domain of variation of  $z$  thus defined will be designated by  $D$ .

In an approach introduced by Philbrick,<sup>36</sup> a finite number of values  $z_k$  is chosen to be reproduced with the required accuracy. All other points on the  $z$  surface are obtained by interpolation. This approach represents a generalization of the familiar diode function generating technique for  $y = f(x)$ . In this case, however, planar surfaces are used in place of the line segments. This implies that the function generator will consist of a two-dimensional array of logic circuits. The domain  $D$  is first divided into rectangular subdomains  $d$ , one of which is shown in Figure 6.30. Note that the diagonal separates the subdomain  $d$  into two parts. In one subdomain  $x < y$ , whereas in the other  $x > y$ . The equation of the diagonal is  $y = x$ , assuming  $\Delta x = \Delta y$ . The simple logic circuit of Figure 6.31 is used to produce an output voltage proportional to the value of whichever of the two inputs is smaller than the other. This circuit will thus generate  $e_a = \min(x - x_i, y - y_j)$ . Having selected one or the other part of the subdomain  $d$ , the diode  $P_3$  in Figure 6.31 will start conducting and approximating the corresponding surface curve by a

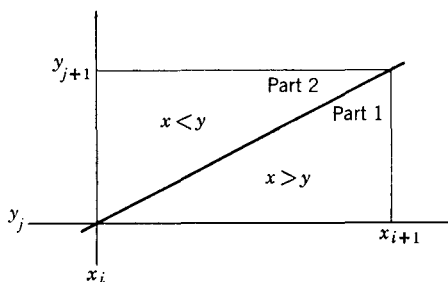


Fig. 6.30 Subdomains  $d$ .

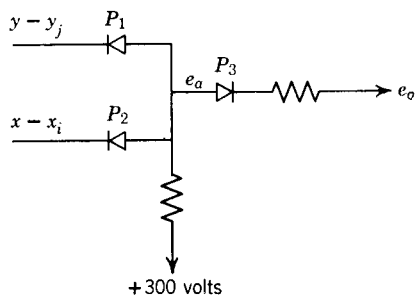


Fig. 6.31 Logic circuit for selecting smaller of two variables.

linear segment. However, this will be true only if  $\min(x - x_i, y - y_j) > 0$ , because otherwise  $P_3$  is biased in a backward direction. This fact can be conveniently expressed by stating that the output of the circuit in Figure 6.31 is

$$e_o = \max [\min (x - x_i, y - y_j) 0] \quad (6.43)$$

where  $\max [\min (x - x_i, y - y_j) 0]$  refers to whichever of the two variables  $\min (x - x_i, y - y_j)$  or zero is greater. Thus, one first selects the smaller of  $x - x_i$  and  $y - y_j$ , and then starts the interpolation only if the selected variable is greater than zero. The biasing of  $P_3$  is necessary to stop the conduction of  $P_3$  whenever  $x - x_i < 0$ , or  $y - y_j < 0$  so that the different subdomains  $d$  can be isolated. In Figure 6.32 the planar surface defined by Equation 6.43 is represented for the subdomain  $d$ . The two triangular facets of this planar surface are characteristic of this approximation. The

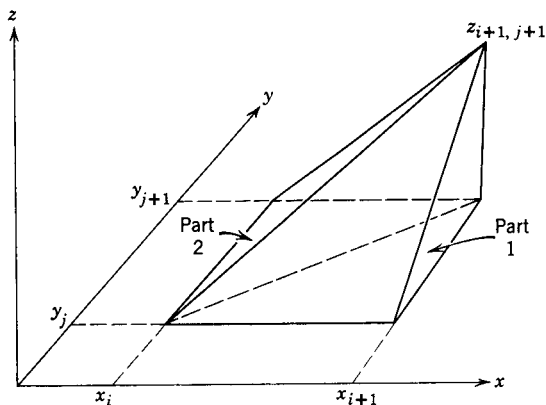


Fig. 6.32 Subdomain  $d$  including  $z$  coordinate.

PREPARATION OF ASSUMED DATA							
		← Z Determined Values/Scaled Equiv's. in volts →					
$y_f$ (+15)	$y_5$	45 (30 v)	114 (76 v)	144 (96 v)	105 (70 v)	138 (92 v)	111 (74 v)
$y_e$ (+10)	$y_4$	30 (20 v)	90 (60 v)	123 (82 v)	114 (76 v)	120 (80 v)	99 (66 v)
$y_d$ (+5)	$y_3$	15 (10 v)	63 (42 v)	96 (64 v)	96 (64 v)	96 (64 v)	81 (54 v)
$y_c$ (0)	$y_2$	0 (0 v)	39 (26 v)	69 (46 v)	81 (54 v)	75 (50 v)	60 (40 v)
$y_b$ (-5)	$y_1$	-15 (-10 v)	18 (12 v)	45 (30 v)	54 (36 v)	48 (32 v)	39 (26 v)
$y_a$ (-10)	$y_0$	-30 (-20 v)	0 (0 v)	24 (16 v)	30 (20 v)	27 (18 v)	21 (14 v)
		$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
		$x_a$ (4)	$x_b$ (8)	$x_c$ (12)	$x_d$ (16)	$x_e$ (20)	$x_f$ (24)

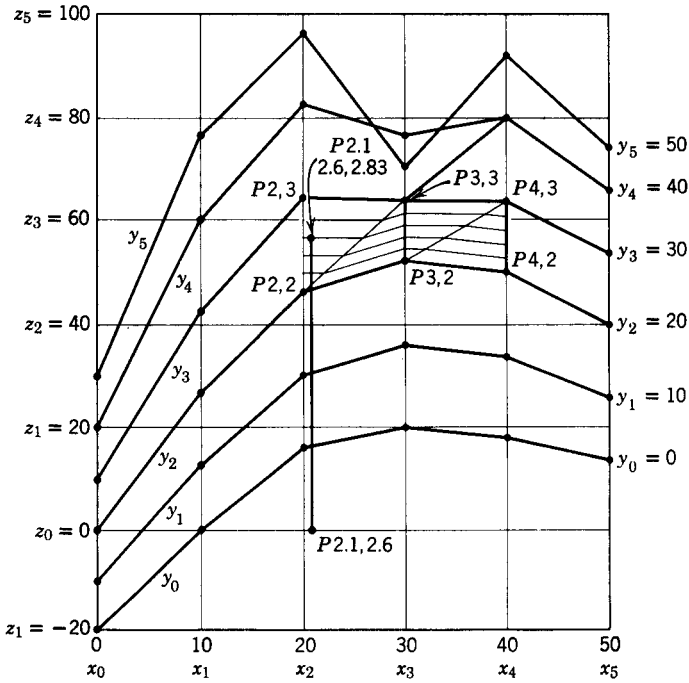


Fig. 6.34 Generation of arbitrary function surface (George A. Philbrick Researches, Inc.).

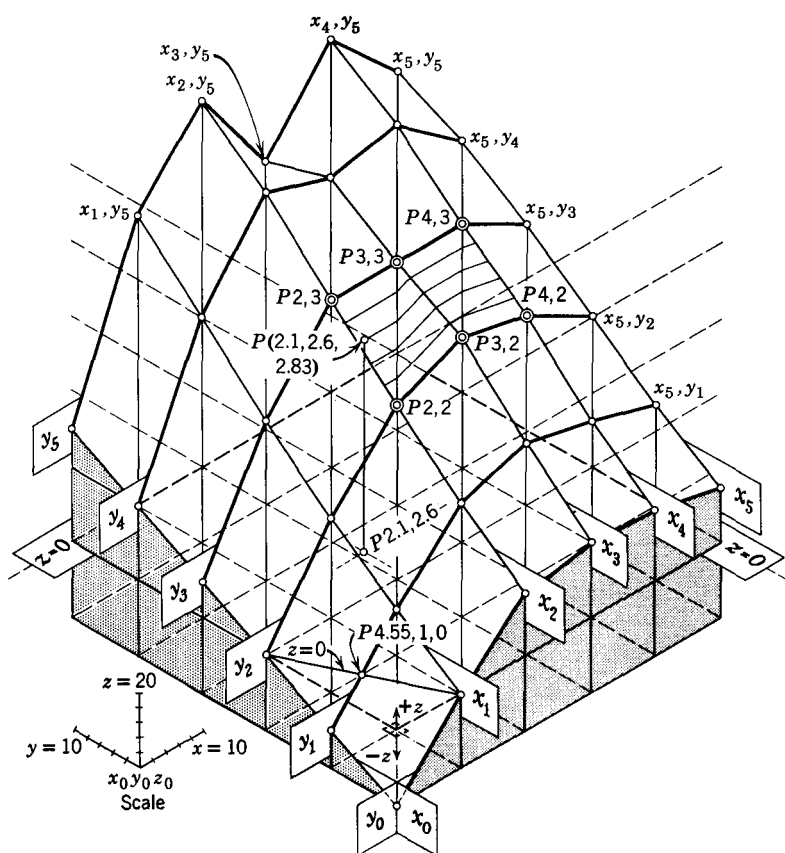


Fig. 6.34 (Continued)

surface is easily constructed by letting  $x = 0$ , and then  $y = 0$ , as well as following the selections dictated by Equation 6.43. In part 1 of subdomain  $d$  in Figure 6.32,  $y = x - x_i$ , so that for a specific  $x = x_a$  the interpolation is carried out along  $z = f(x_a, y)$ . The opposite is true in part 2, so that  $z = f(x, y_b)$ .

Starting from the basic planar surface of Figure 6.32 any curvilinear surface  $z = f(x, y)$  can be approximated. The general expression for such an approximation is

$$z(x, y) \approx [z(x, 0) + z(0, y) - z(0, 0)] + \sum_{i=1}^n \sum_{j=1}^m z_{ij} \max [\min (x - x_i, y - y_j), 0] \quad (6.44)$$

Starting from given  $z(0, 0)$ ,  $z(x, 0)$ ,  $z(0, y)$ , the required  $z_{ij}$  can be reached by adjusting the slopes of diodes ( $P_3$  in Figure 6.31) at various points. The detailed circuit is shown in Figure 6.33 for  $i = 1, \dots, 6, j = 1, \dots, 6$ . The network has a matrix of 36 elementary diode selection circuits of the type shown in Figure 6.31. The correspondingly biased operational amplifiers serve for the separation of the input voltages  $x, y$  into  $x_{i+1} - x_i, y_{j+1} - y_j$  slices. Very simple operational amplifiers having gains of the order of 100 are adequate for this purpose. A practical example of this approximation is shown in Figure 6.34 together with the set of initial data and the surface obtained. Kehr<sup>37</sup> has effected an important improvement in this technique. The values of  $z(x, y)$ , instead of being adjusted as voltages and summed as such, are obtained as increments of the slope of  $z$  and summed as currents. Thus all settings of  $z$  are completely independent of each other.

In the approach just described the formation of the desired surface is obtained by the superposition of the pyramidal elements of Figure 6.32. Petternella and Ruberti<sup>38</sup> have developed a method of generating a series of planar surfaces and using them to approximate  $z = f(x, y)$ . This represents a direct extension of the one variable linear segment approximation with all its generality and flexibility. The circuit shown in Figure 6.35 is used to generate the planes. The output voltage  $e_o$  is

$$e_o = \frac{e_{i1}G_1 + e_{i2}G_2 + EG_E + VG}{G_1 + G_2 + G_0 + G + G_E} \quad (6.45)$$

where  $G_i$  are conductances. Equation 6.45 is that of a plane intersecting  $e_o = 0$  along the line

$$e_{i1}G_1 + e_{i2}G_2 + EG_E \pm VG = 0 \quad (3.46)$$

and having a slope

$$p = \frac{\sqrt{G_1^2 + G_2^2}}{G_1 + G_2 + G_0 + G + G_E} \quad (6.47)$$

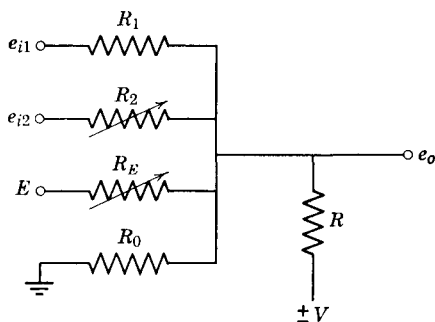


Fig. 6.35 Circuit to generate planes:  $z = ax + by + c$ .

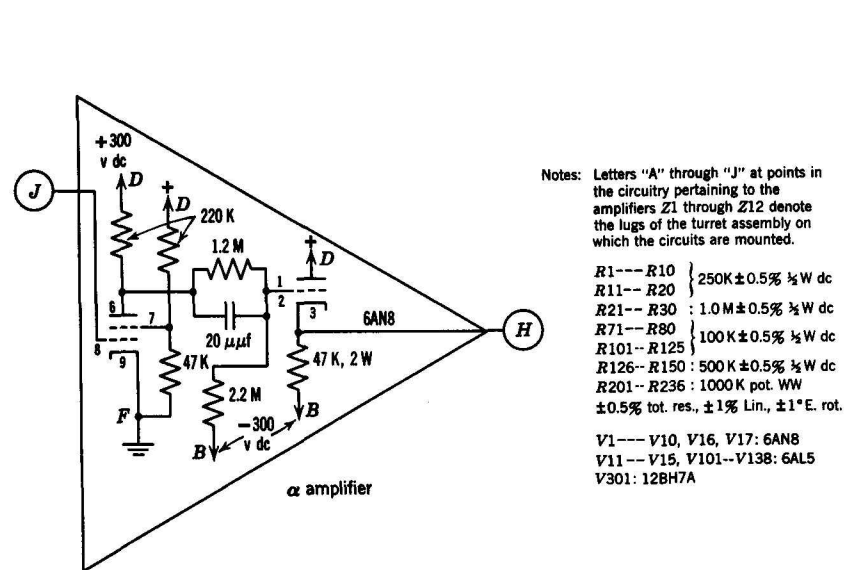
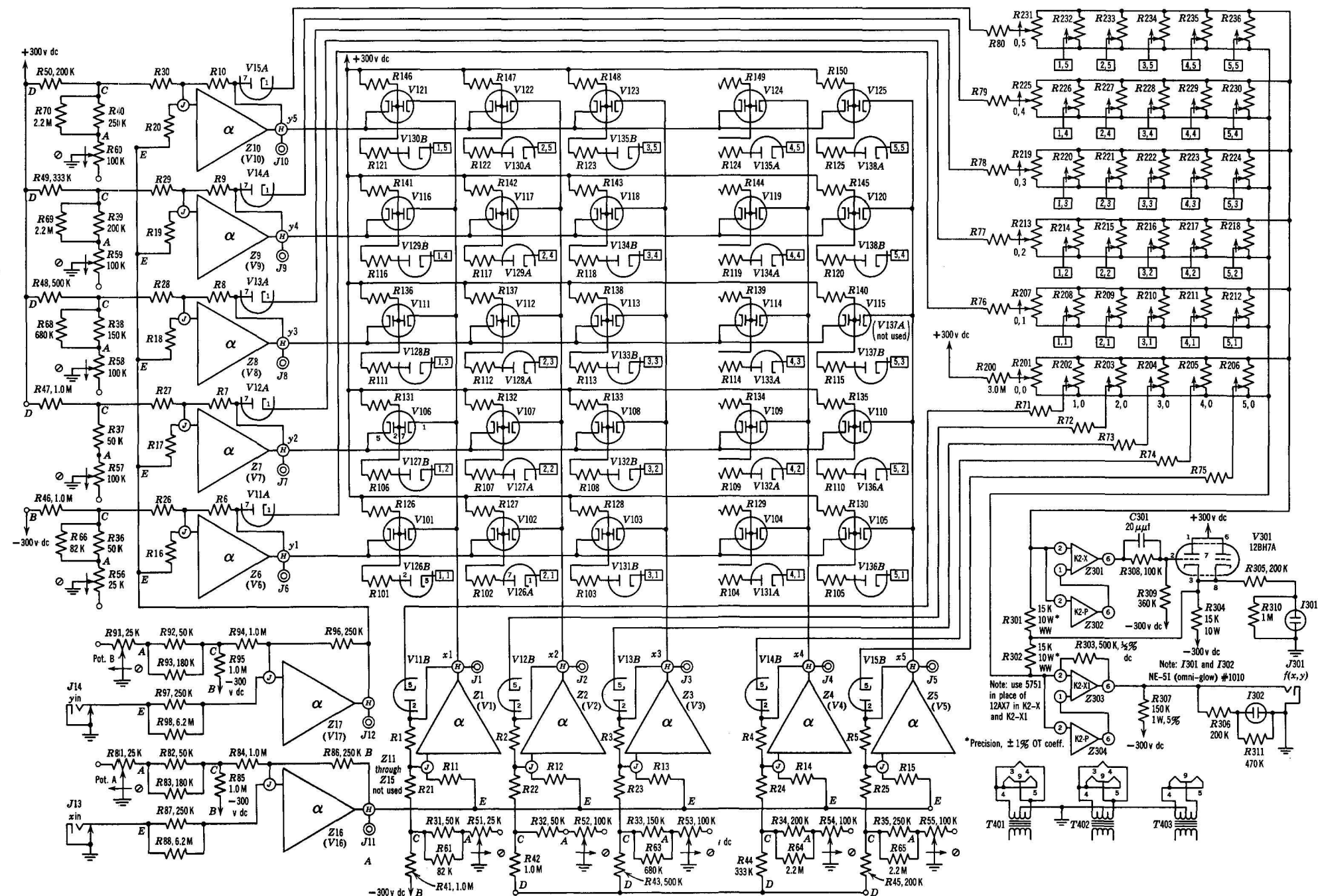


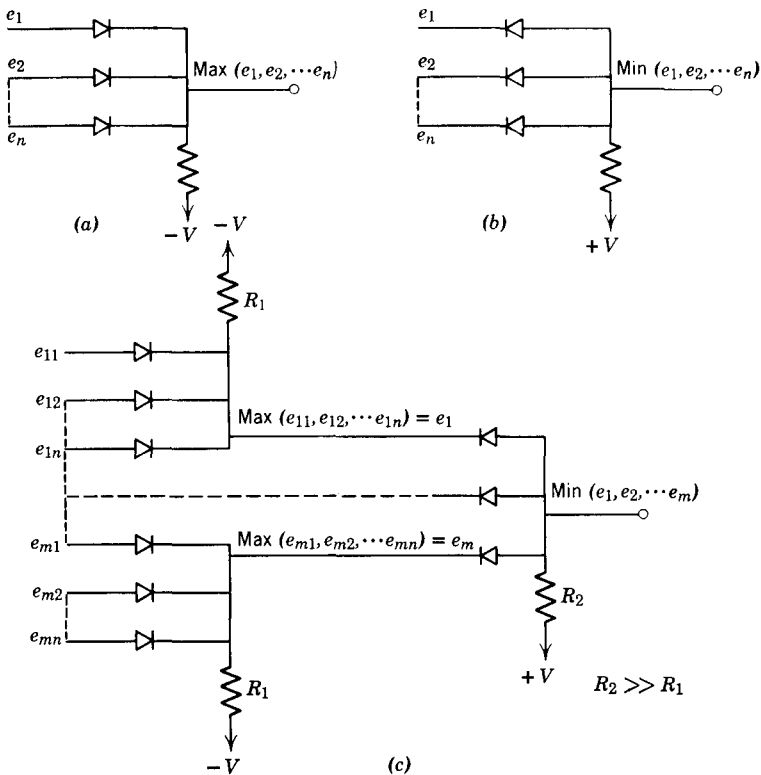
Fig. 6.33 Circuit diagram of generator of functions of two variables (George A. Philbrick Researches, Inc.).



Now by using  $e_{i1}$ ,  $e_{i2}$  either directly or inverted, as well as the positive or the negative reference voltage  $E$ , the eight types of approximating planar surfaces can be obtained. These correspond to the eight types of linear segments for curve approximation, as indicated in Figure 6.16. The procedure employed to set up the function generator is similar in principle to that used for one variable. For a given  $z = f(x, y)$  the approximating planes are calculated in the form

$$z = ax + by + c \quad (6.48)$$

The voltage  $\pm V$  of Figure 6.35 is used for diode biasing to permit the selection of the appropriate plane. The logic circuits used for this purpose are represented in Figures 6.36a and b and perform the same minimum or maximum operation as before to select the plane that gives a maximum or minimum value of output voltage respectively. Figure 6.36c shows a circuit for double selection. Evidently the accuracy of the approximation



**Fig. 6.36** (a) Logic circuit for selecting maximum of  $n$  variables. (b) Circuit for selecting minimum of  $n$  variables. (c) Logic circuit for double selection.



depends upon the number of planes used. The device described consisted of two units each having ten planes.

A mathematically different approach has been used by Meissinger.<sup>39</sup> The function  $z = f(x, y)$  is represented in parametric form  $z = f(x, y_k)$ ,  $k = 1, 2, \dots, n$ , so that the generator has to produce a family of curves with a variable parameter. The parametric method implies two sweeping speeds, one for  $x$  and another for  $y$ . The advantage of this method is that one-variable function generators can be used. To apply this method, isoclines of the family of curves  $z = f(x, y_k)$  must be drawn. A graphical method has been developed to obtain the parametric functional relation to drive the breakpoint reference voltage.

Elliott<sup>40</sup> employs a similar technique for matching families of curves having reasonably regular characteristics. His method involves the use of a diode function generator to match accurately one of the curves of the family. This curve then represents the function of one of the variables. By the addition of suitable voltages to the input and the output of this function generator, the basic curves can be translated in the horizontal or vertical directions. The amount of translation is controlled by the second input variable. Combined horizontal and vertical translations are achieved by adding voltages to the input as well as to the output of the function generator. More sophisticated versions of this technique make it possible to rotate as well as to translate the basic reference curve in order to permit the approximation of a wider variety of families of curves. This makes it unnecessary for the curves to be represented by sets of parallel lines as in Meissinger's method.

The universal function generator described in the preceding section can be adapted to the generation of functions of two variables.<sup>41</sup> This is accomplished by introducing one of the functional relationships to the circuits determining the comparison level of the amplitude comparators. The other functional relationship is introduced as before as settings of potentiometers  $P_1 \dots P_n$ .

Several methods have been proposed for the utilization of photographic plates for the storage of the function to be generated. Two such devices have been proposed by Wallman<sup>42</sup> and Wentzel.<sup>43</sup> In both these devices, one of the independent variables must be time. The function  $K(x_i, t)$  is stored on a photographic plate. Thirty columns are used for discrete values of  $x$ , whereas  $t$  is maintained in continuous form. Figure 6.37 shows the method of film preparation. The width of each transparent column represents the function  $K(x_i, t)$ . The columns are scanned by the light beam from a cathode-ray oscilloscope, and the light pulses are sensed by a photocell. During each minor cycle, the range of the variable  $t$  is scanned. After each minor cycle, the process is repeated for the

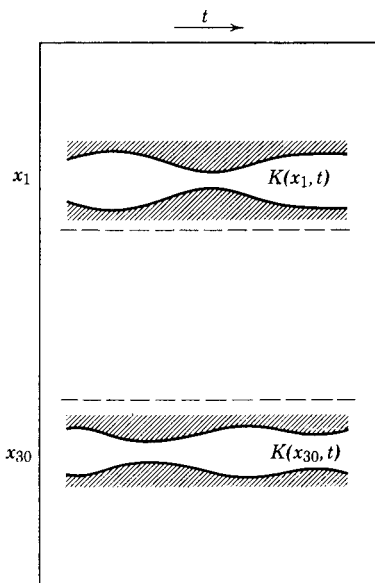


Fig. 6.37 Photographic plate used for function generator.

succeeding value of  $x_i$ . The scanning of each column requires 75 msec, so that the entire function is scanned in 3.5 sec. The accuracy of such a device is reported to be of the order of 1%.

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